Slab Delivery of Incoherent Pump Light to Double-Clad Fiber Amplifiers: An Analytic Approach

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***Abstract*—Delivery of incoherent diode bar pump power via a tapered slab into a double-clad fiber amplifier allows for compact and efficient devices. We provide analytic estimates of system parameters for robust slab coupled configurations.**

***Index Terms*—Double-clad fibers, optical fiber amplifiers, optical fiber lasers, tapering.**

I. Introduction

DOUBLE-CLAD fiber amplifiers are widely used to convert the multimode pump power of diode bars or stacks into single-mode signals [1]–[4]. Impressive power scaling of such devices has already been demonstrated [4]–[6]. Many applications still require multi-kilowatt to megawatt scaling of power while retaining high brightness. Among the various problems encountered at such power scaling levels are the processes of unwanted stimulated scattering [4], [7], [8] and thermal or critical self-focusing damaging. Overcoming the stimulated Brillouin scattering problem will open the doorway to a new generation of lasers and important applications [9]. One of the ways to mitigate the stimulated scattering is to reduce the length of the amplifier.

To make the amplifier short and efficient, the effective absorption rate of the pump should be high. The local pump absorption rate in the core is limited by the concentration of active centers in the fiber; for typical glass it does not exceed 10 cm^{-1}. Also, the efficiency of the double-clad fiber depends on its design. The precision of adjustment of the step of the refraction index and inhomogeneities of the glass limit the size of the single-mode core in fiber amplifiers. Usually, the radius of the core does not exceed 10 μm. On the other hand, the cladding should be large enough in order to capture the incoherent pump light. It is important to make all of this pump available for the absorption in the core. Therefore, the pump should be effectively mixed within the cladding. Various shapes of cladding were compared [10]–[15] to improve this mixing. Even with good mixing of the pump, the effective absorption rate is limited by the product of the local absorption rate to the ratio of area of the cross section of the core to that of the cladding [14]–[18]. How do we reduce the cladding area if its width is determined by the beam parameters of the diode pump source and its thickness has to be greater than the diameter of the core? Currently, beam-coupling optics is used in order to match the pump spot to the fiber inner-cladding shape [13], [19]. This introduces additional losses and does not address the problem of coupling incoherent pump light over short distances.

Recently, we suggested a tapered-slab delivery scheme to couple a diode bar pump into the fiber [20], [21].[1] In this case, the conventional wide cladding is replaced by a narrow slab that is in direct contact with the core and has no need to evolve it. This scheme takes advantages of the enormous aspect ratio of beam provided by typical sources of pump and allows direct butt-coupling to the laser diode bar.

However, special efforts are necessary to make such a fiber laser single-mode. The problem is that the slab should have an index of refraction that is much higher than that of the surrounding medium (outer cladding); and if the index of refraction of the wide core is similar or higher than that of slab, then the core supports propagation of many modes. The solution suggested is that unwanted modes of the signal leak into the slab; then indices of refraction of the core and cladding can be high. Such a solution puts some constraints on the relative refractive indices of the various layers in the structure. The index of refraction of the surrounding medium (outer cladding) should be much lower than that of the slab, in order to confine the multimode pump. The index or refraction of the slab itself should be slightly greater than that of the core. This condition confines the principal mode of the signal to the core, and the higher order signal modes leak into the slab, filling the whole waveguide. To make such a device efficient, the index of refraction of the slab should be adjusted within an error of 1%, even at moderate values of core radius. Such precision places constraints on manufacturability, especially if the core is supposed to be large.

Here we analyze an alternative configuration, which does not require precise adjustment of parameters of the slab. Let the core be surrounded by a relatively narrow cladding, which is sufficient to prevent the principal mode of the signal from leaking into the slab (Fig. 1), even if the refractive index of the slab is high. The left side of such a device looks very much like a conventional single-mode fiber. The right-hand side of Fig. 1 rep-

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[1]Kano indicated an error in these papers: [21, fig. 9] corresponds to indices of refraction $n_2 = 1.58$, $n_3 = 1.54$ and not to $n_2 = 1.57$, $n_3 = 1.55$ as printed there. Notations n_2, n_3 of [21] correspond to n_3, n_4 in this paper.

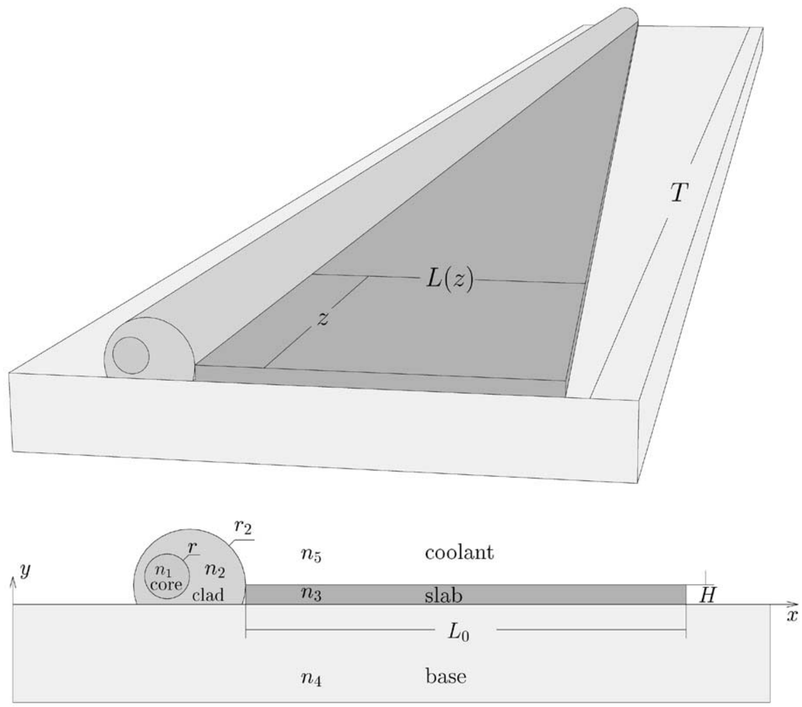

Fig. 1. Waveguide with tapered-slab delivery of pump into the double-clad fiber.

resents the tapered slab; its width depends on the longitudinal coordinate z. The area of the cross section of the cladding can be small in comparison to that of the slab; then the effective absorption rate of the pump is similar to that in the slab-core geometry [20], [21] or even greater, as the single-mode core can be larger.

In a tapered-slab waveguide, the highest order lateral pump modes of the diode bar can leak away. For effective absorption of the pump, it is important to absorb the highest modes quickly in the doped core before they are lost. Therefore, the highest modes of the slab should be in resonance with lowest modes of the fiber. Such resonance implies that the lowest modes of the slab are uncoupled from the core, at least at the beginning of the amplifier. Such a decoupling does not reduce the efficiency of the device because the intermediate and lowest modes of waveguide are pushed into the core from the slab further along the taper. This brings them into resonance with modes of the fiber and leads to their effective absorption by the core. Due to the essentially multimode character of the pump, the index of refraction of the slab has no need to be adjusted precisely as in the slab-core geometry as long as the single-mode propagation of the signal in the core is sustained by the cladding.

At first glance, delivery of the pump with the tapered slab is similar to the nonfused fiber coupler [22], which still requires large cladding. The main difference is that, with the tapered-slab delivery, the fiber has no need to support propagation of many modes of the pump, as the core absorbs them as they penetrate the fiber. The remaining nonabsorbed modes of the pump are expulsed from the waveguide. An appropriate match of slab and fiber parameters allows for a drastic reduction of the cross section of cladding and, therefore, the required length of the amplifier. However, we need to estimate how wide the cladding should be in order to confine the signal within the core, how high an index of refraction of the slab is needed to provide strong penetration of the highest modes of the pump into the fiber at the beginning of the taper, and how gentle the tapering needs to be in order to avoid expulsion of these modes from the waveguide before the core absorbs most of their power.

II. Quasi-Guided Mode and Rate of Its Escape

This section estimates the thickness of cladding necessary to confine the principal mode of the signal. Before we deal with realistic cases, consider first the principal mode of a circular fiber with wide outer cladding of high refractive index. Following [23] and [24], consider the scalar equation

$$\Delta_\perp \psi + \left({k_0}^2 n(x,y)^2 - \beta^2 \right) \psi = 0 \tag{1}$$

where $\Delta_\perp$ is the transversal Laplacian, k_0 is the vacuum wavenumber of the signal, $n(x,y)$ describes the distribution of the index of refraction, and β is the propagation constant. Using cylindrical coordinates, we define the index of refraction as follows:

$$n(\rho) = \begin{cases} n_1, & \rho < r_1 \\ n_2, & r_1 < \rho < r_2 \\ n_3, & r_2 < \rho \end{cases} \tag{2}$$

While $n_3 > n_1 > n_2$, the quasi-guided mode escapes into the outer cladding, and we need to find conditions such that the part which escapes is small along the length of the amplifier.

Assuming weak coupling, we specify that the escaping mode satisfies $\psi(r_2) = 0$. This boundary condition is analogous to the case of an infinitely strong step of the refractive index between cladding and outer cladding. However, in the case of evanescent modes, this condition does not mean that the mode vanishes outside the cladding (such an assumption would be counterintuitive); the evanescent mode which passes through zero has a

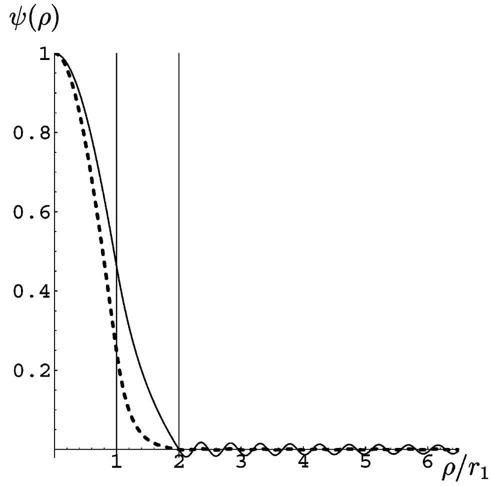

Fig. 2. Two circular evanescent modes. Solid curve: the radial mode ψ using (3) for the case $k_0 = 2\pi/(1.535\ \mu)$, $r_1 = 9\ \mu$, $r_2 = 18\ \mu$, $n_1 = 1.56$, $n_2 = 1.559$ ($V = 2.06$), $n_3 = 1.6$. Dashed curve: the same but $n_2 = 1.555$ ($V = 4.6$).

smaller "tail" than a mode which has a significant value at the boundary of cladding.

To estimate the escape rate at finite values of this step, we extend the solution into the outer cladding and write the principal mode as

$$\psi(\rho) = \begin{cases} J_0(p_1\rho), & \rho < r_1 \\ \mathcal{A}\left(K_0(q_2\rho) - \frac{K_0(q_2 r_2)}{I_0(q_2 r_2)} I_0(q_2\rho)\right), & r_1 < \rho < r_2 \\ \mathcal{B}J_0(p_3\rho) + \mathcal{C}Y_0(p_3\rho), & r_2 < \rho \end{cases} \tag{3}$$

where J, Y, K, and I are Bessel functions. Two examples of such solutions are plotted in Fig. 2.

Six parameters ($\mathcal{A}, \mathcal{B}, \mathcal{C}, p_1, q_2, p_3$) appear in (3). Substitution of (3) into (1) gives relations for q_2 and p_3: $q_2{}^2 = k_0{}^2(n_1^2 - n_2^2) - p_1{}^2$ and $p_3{}^2 = p_1{}^2 - k_0{}^2(n_3^2 - n_2^2)$. The smoothness of the mode (continuity of derivatives) gives four additional relations to calculate the coefficients $\mathcal{A}, \mathcal{B}, \mathcal{C}$, and transversal wavenumber p_1. The product of the latter with the core radius gives the dimensionless transversal wavenumber $P = p_1 r_1$. This parameter can be treated as function of two arguments, $R = r_2/r_1$ and $V = k_0 r_1\sqrt{n_1^2 - n_2^2}$, and tabulated or fitted once for all circular fibers.

From Fig. 2, we see that the tail of the evanescent mode has decayed for moderate values of the ratio of radii r_2/r_1 and normalized frequency V. We would expect a small escape rate for such a mode. To estimate this escape rate, consider the asymptotic behavior of the mode (3) for $\rho > r_2$ as follows:

$$\psi(\rho) \approx a\sqrt{\frac{r_2}{\rho}}\exp(ip_3\rho) + b\sqrt{\frac{r_2}{\rho}}\exp(-ip_3\rho). \tag{4}$$

We assume fast oscillations of this tail (see Fig. 2), so, the Bessel functions from (3) are replaced here with their asymptotic behavior. Note that $|a| = |b| = |\psi'(r_2)/(2p_3)|$. The first term on the right-hand side of (4) can be treated as an outgoing wave. Its

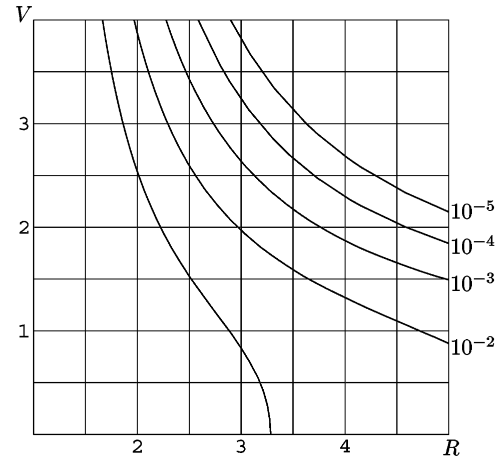

Fig. 3. Contour plot of normalized escape rate $E(R,V)$ using (5). Levels 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, and 10^{-5} are shown. For the case of (7), the plot represents the effective escape rate $\mathcal{K}_{\text{eff}}$ measured in m^{-1}.

wave vector is inclined at an angle $\vartheta = p_3/(nk_0)$ to the optical axis; this wave carries away flux as follows:

$$F = 2\pi\rho\left|a\sqrt{\frac{r_2}{\rho}}\right|^2 \vartheta = 2\pi r_2|a|^2\,\vartheta = \frac{\pi r_2\,|\psi'(r_2)|^2}{2p_3 n_3 k_0}.$$

The second term in (4) corresponds to a wave going toward the core. Assume that this wave penetrates well into the core. Then its absence does not affect, in the first approximation, the outgoing wave. Let $\|\psi\|^2 = 2\pi\int_0^{r_2}|\psi(\rho)|^2\rho\mathrm{d}\rho$ be the squared norm of the mode. Then we can estimate the escape rate as follows:

$$\mathcal{K} = \frac{F}{\|\psi\|^2} = \frac{p_3}{n_3 k_0}\frac{2\pi r_2\left(\frac{1}{2p_3}\psi'(r_2)\right)^2}{2\pi\int_0^{r_2}\psi(\rho)^2\rho\mathrm{d}\rho} = \frac{RE}{D} \tag{5}$$

where $D = 4k_0 n_3 p_3 r_1^3 \approx 4k_0{}^2 r_1^3\sqrt{n_3^2 - n_1^2}$ and $E = (r_1{}^4|\psi'(\rho)|^2)/(\int_0^{r_2}|\psi(\rho)|^2\rho\mathrm{d}\rho) = E(R,V)$ is a function of its two arguments only. A contour plot of this function is shown in Fig. 3. This function decreases exponentially with an increase of the product RV. Typical values of length D are of the order of centimeters, so, at $RV > 6$, the escape rate is the order of $1/m$ or less.

For the realistic case with broken symmetry (Fig. 1), the escape rate should be even smaller than in the estimate above. The quasi-guided mode can leak neither into the substrate nor into the coolant. It can only escape in directions with total angle $\theta = H/\tilde{r}_2$, where H is the thickness of the slab and $\tilde{r}_2$ is the distance from the center of the core to the slab. This distance can be slightly greater than the radius r_2 of the truncated circle which represents the cross section of cladding in the bottom part of Fig. 1. Assuming that the substrate does not strongly perturb the mode, then the effective escape rate should be approximately proportional to the angle θ. This gives the estimate for the escape rate as

$$\mathcal{K}_{\text{eff}} = \mathcal{K}\frac{\theta}{2\pi} = \frac{E(R,V)}{\ell}, \quad \ell = \frac{8\pi n_3 k_0^2 r_1^4\sqrt{n_3^2 - n_1^2}}{H}. \tag{6}$$

This is the main result of this section. The effective rate of escape is expressed in terms of parameters of the waveguide. For example, for

$$r_1 = 10\,\mu\text{m},\;\; H = 4\,\mu\text{m},\;\; n_3 = 1.6,\;\; n_3 - n_1 = 0.11,\\ k_0 = \frac{2\pi}{1.535}\mu\text{m} \tag{7}$$

we get $\ell = 1$ m. Then, Fig. 3 can be interpreted as a contour plot of the escape rate $\mathcal{K}_{\text{eff}}$ measured in m^{-1}.

Reasonable values of parameters R and V that yield a robust structure are not sensitive to the length of the amplifier. For typical cases, the product RV should be of the order of six. For example, at $V = 2$, the distance between the core and slab should be of the order of the diameter of the core. Then, on the one hand, the area of cladding is small in comparison to that of the slab and, on the other hand, the signal almost does not escape over the first meters of the amplifier. The estimate (6) is still overstated. In the vicinity of the slab, the low-refractive-index substrate reduces the amplitude of the mode relative to the symmetric distribution assumed in deriving (3), so the rate of escape of the signal should be even less than the estimate (6) predicts. We see that a relatively small cladding sustains the propagation of the signal.

The configuration with a slab pump of the fiber amplifier is robust. The values suggested above do not change much if we vary the thickness of the slab, its refractive index, and/or the length of the amplifier. The length of device required for the effective absorption of the pump in such a device is estimated in the following section.

III. Optimal Tapering

Assume that the dimensions of the cladding are chosen according to the procedure of the previous section. Then, for a given area of cross section of the waveguide, we can estimate the length of the amplifier required for the efficient absorption of the pump delivered by the tapered slab.

Let the index of refraction step from core to cladding be small. This may correspond to a large single-mode core. Then most of the incoherent pump light coming from the slab should not feel this step. This allows for the approximation of ergodic mixing, i.e., the intensity of the pump is approximately constant over the cross section of the waveguide, and the effective absorption rate can be estimated as the product of the local absorption rate to the ratio of area of core to the total area of waveguide [13], [21]–[23] as follows:

$$\alpha_{\text{eff}}(z) = \alpha \frac{\pi r_1^2}{S + HL(z)} \tag{8}$$

where S is the area of the cladding, $L(z)$ is the width of the tapered slab at a distance z from the beginning, and α is the local absorption rate of the pump in the core. In general, α may vary due to saturation, but in this analysis we assume it to be constant. Then the differential equation for the pump power $W(z)$ can be written as follows:

$$W'(z) = -\alpha_{\text{eff}}(z) W(z). \tag{9}$$

Consider the linear taper; let $L(z) = L_0(T - z)/T$ where T is the length of the amplifier and L_0 is the initial width of the slab. Dividing both sides of (9) by $W(z)$ and integrating the result with respect to z, we obtain

$$\frac{W(z)}{W(0)} = \left(1 - z\frac{HL_0}{(HL_0 + S)T}\right)^{\frac{\alpha\pi r_1^2 T}{HL_0}}. \tag{10}$$

Efficient pumping conditions are realized when the intensity of the pump is uniformly distributed along the core. Then the intensity of the pump does not drop along the amplifier. This allows the active medium to work in the most efficient manner along the whole amplifier. The uniform intensity of the pump in the core means a linear dependence of the pump power on z. To realize this linear dependence, we look at (10) and choose

$$T = \frac{HL_0}{\alpha\pi r_1^2}. \tag{11}$$

Then, the efficiency of absorption of the pump can be estimated as follows:

$$\frac{W(0) - W(T)}{W(0)} = \frac{HL_0}{HL_0 + S}. \tag{12}$$

If S is small in comparison to the initial area HL_0, then the efficiency approaches unity. For example, at $\alpha = 10\ \text{cm}^{-1}$, $r_1 = 10\ \mu$m, $H = 6\ \mu$m, $L_0 = 1$ cm, and $S = 1000\ \mu\text{m}^2$, we get most of the pump light absorbed at a length $T \approx 20$ cm. In the case of a conventional double-clad device, the corresponding length would be much larger, mainly due to the large area of the inner cladding. A tapered-slab delivery of the pump gives a means of reducing the length of the amplifier while retaining high pump efficiency.

The solution above is self-consistent only in the case of a linear dependence of $W(z)$. With strong tapering (small values of T), the pump just leaks away and the derivation above does not take this into account. For a highly doped core, it may make sense to make T slightly larger than the estimate (11) suggests. This case is considered in Section V. However, in this case also, the dependence of $W(z)$ remains almost linear because, even with good coupling, the core cannot absorb the pump faster than the slab delivers it. The conditions of good coupling are analyzed in the next section.

IV. Indices of Refraction

The previous section ignores leakage of the pump from the tapered waveguide. Therefore, the estimate (12) is overstated. To reduce this leakage, the indices of refraction n_4 and n_5 (Fig. 1) should be as small as possible. This leakage is effectively suppressed if each mode of the pump is strongly absorbed in the core before to leave. Strong absorption takes place when a mode of the slab passes through resonances with modes of the cladding. In order to force each mode of the pump to pass through all such resonances, the index of refraction n_3 of the slab should be adjusted according to the numerical aperture of pump in the x direction. In this section, we estimate the required value of n_3.

Adiabatic tapering of the slab increases the transversal wavenumber of the modes, reducing the propagation constant of a particular mode of the slab. At some propagation length, this propagation constant coincides with that of some mode of the cladding and a resonance occurs. The cladding is multimode, so each mode passes through many such resonances along the taper and we have no need to tune a mode of the slab to any specific mode of the cladding. However, there exist some constraints on the index step $n_3 - n_2$. Too high a value of n_3 isolates the pump from the fiber, leaving the beginning of the core unpumped. Too low a step of the refractive index means that the highest modes miss many resonances with the cladding, and these modes may escape before they are efficiently absorbed in the core. Therefore, the amplifier should use this part of the pump quickly, while it is still confined in the waveguide. At the beginning of the taper, the highest modes of the slab should be in resonance with the lowest modes of the cladding. This means that the highest modes of the pump barely penetrate the cladding.

The slab is wide, thus its modes can be treated as plane waves in the xz plane or even as rays of geometric optics. Let NA be the initial angle of divergence of the pump in the x direction. This angle is determined by the source of pump. Consider the plane wave (or ray) corresponding to the wave packet formed from highest modes of the pump. This wave is incident to the cladding at the angle $\pi/2 - \mathrm{NA}/n_3$. Let this be the critical angle. Then, the corresponding refractive index of the slab can be estimated from the Fresnel refraction law: $n_2 \sin(\pi/2) = n_3 \sin(\pi/2 - \mathrm{NA}/n_3)$. In the paraxial approximation ($\mathrm{NA} \ll 1$), we obtain the estimate

$$n_3 = \frac{n_2}{\cos\frac{\mathrm{NA}}{n_3}} \approx \frac{n_2}{1 - \frac{1}{2}\left(\frac{\mathrm{NA}}{n_3}\right)^2} \approx \frac{n_2}{1 - \frac{1}{2}\left(\frac{\mathrm{NA}}{n_2}\right)^2} \approx n_2 + \frac{\mathrm{NA}^2}{2n_2}. \tag{13}$$

For example, at $n_2 = 1.56$, $\mathrm{NA} = 0.1$, (13) gives $n_3 \approx 1.563$. As the pump should penetrate the fiber at the beginning of the tapering, (13) slightly overestimates the optimal refractive index of the slab. However, instead of adjusting the index of refraction of the slab, the adiabatic pretapering of the slab can be used to make NA slightly greater than $\sqrt{2n_3(n_3 - n_2)}$ at the beginning of the amplifier.

Condition (13) allows an interpretation in terms of geometrical optics. The modes with numerical aperture NA can be treated as rays, inclined at an angle NA/n_2 to the optical axis. Then, the estimate above should work also for other geometries with lateral injection of the pump into multimode fibers, for example, nonfused coupler [22]; good coupling takes place when the pump is delivered with a tapered waveguide of high refractive index. It should be so high that the highly inclined rays of the pump are incident on the fiber at the angle close to the critical angle. This mechanism opens a way to effective and compact powerful amplifiers.

V. Limits of Validity of the Analytical Estimates

The estimates above assume that the pump is well mixed and its intensity does not vary much within the cross section of the waveguide. Then appropriate tapering provides a uniform intensity of the pump in the core along the amplifier. However, the pump needs some length in order to reach the core. If the width of the slab is compatible with the transversal size of the cladding, this length may be compatible with the length of the tapering, especially in the case of a highly doped core which ends at the end of the tapering. Then it is important that the pump is injected into the fiber with some reasonable transversal wavenumber, and (13) may slightly overestimate the optimal index of refraction of the slab. Also, the high absorption of the pump may deplete the pump in the center of the core. Then the estimates above become too optimistic. Consider the case below.

The lowest modes of the fiber have transversal wavenumber of the order of $1/r_2$ where r_2 is the size of the cladding. (The cladding has no need to be circular, but for the estimate let r_2 be its effective transversal size.) These modes come into resonance with the slab modes, providing the efficient absorption of the pump. The lowest cladding modes correspond to rays inclined to the optical axis with angle $\Theta = 1/(kr_2)$ where $k = 2\pi n_1/\lambda$. Going to the center of the core, such a ray reduces its intensity $\exp(-\alpha r_1/\Theta)$ times. We can define a parameter α_0, some reference absorption rate, such that $r_1\alpha_0/\Theta = 1$; this gives $\alpha_0 = 1/(r_1 r_2 k)$. For example, at $r_1 = 10\ \mu$m, $r = 25\ \mu$m, $k = 10/\ \mu$m, and the reference absorption rate $\alpha_0 \approx 4\ \mathrm{cm}^{-1}$. At $\alpha \ll \alpha_0$, the absorption does not disturb the uniform distribution of the intensity of the pump on the cross section of waveguide, and the estimates (10)–(12) are valid. For $\alpha \gg \alpha_0$, the pump intensity at the center of the core is lower than in the rest of waveguide. Then the length of tapering should be increased in comparison to (12) in order to keep high efficiency of absorption of the pump.

The recent simulations [25] show that, generally, in order to obtain high efficiency, the length T of the tapering should be slightly longer than (11) suggests, but the correction factor is of the order of unity and slowly depends on the ratio α/α_0. However, the slight adjustment of parameters of the slab in comparison to (11) and (13) leads to increase of efficiency in comparison to (12). Equations (11) and (12) allow a quick estimate of reasonable values of parameters for the tapered-slab delivery of the pump to the double-clad fiber amplifier.

VI. Discussion and Conclusion

We have shown that tapered-slab delivery of incoherent pump light from diode bars into a fiber amplifier (Fig. 1) is effective. Our geometry allows amplifier lengths much shorter than those in conventional schemes. This is important for high-power devices where stimulated scattering reduces the performance. For efficient delivery of the multimode pump, the index of refraction of the slab should be higher than that of the core. A relatively narrow cladding preserves the principal mode from leaking into the slab. This configuration does not require precise adjustment of the refractive index or thickness of the slab. In realistic cases, at normalized frequency $V \approx 2$, the distance from the slab to the core should be of the order of the diameter of the core. An appropriate tapering of the slab realizes efficient mixing of the pump, and (11) suggests the optimal length of the taper (and amplifier) at a given pump absorption rate and given ratio of area of core to the initial area of waveguide. This length is similar to that in the case simulated recently of an amplifier with a stripped core [20], [21]. For efficient and uniform pumping of the core, the highest modes of the pump should barely penetrate the fiber at the beginning of the taper. These modes can be treated as plane waves or rays coupled to the fiber at an almost critical angle. The corresponding refractive index of the slab is estimated by (13).

The results above are essentially analytical and general. The numerical check [25] qualitatively confirms the estimates suggested here and shows that slight adjustment of parameters of the slab may even increase the efficiency in comparison to asymptotical estimate (12).

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