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NONLINEAR QUANTUM OPTICS

Self-Modulation of Optical Pulses in a Kerr Medium and Limits of the Single-Mode Approximation[1]

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Abstract—The formal definition of the effective oscillator is suggested. The propagation of an optical pulse in a Kerr medium is formulated in terms of the effective oscillators. Limits of the single-mode approximation are discussed. It is shown that the number of photons escaped from the principal mode to the highest modes is much greater than the square of the nonlinear phase shift of the principal mode. This prohibits the production of the "Schrödinger cat" states by nonlinear automodulation of the coherent rectangular optical pulses in a Kerr medium, but allows the squeezing.

INTRODUCTION

Any nonlinear interaction, in principle, can produce a nonclassical state. The Kerr nonlinearity seems to be the simplest one. Since papers [1–7], it seemed that as the squeezed states, as the *Schrödinger cat* ones can be produced by the self-modulation, it is enough to let the coherent pulse propagate in a Kerr medium. The interaction Hamiltonian $(\hat{a}^{\dagger})^2\hat{a}^2$ really converts the initial coherent state into states with beautiful quasi-distributions, which show the deformation of the uncertainty body (the squeezing), its diffusion and the fractional revivals of quasi-classical wave packets.

At least the squeezing can be observed with the homodine detection scheme as it is shown in Fig. 1. Such experiments are described in [8]. (See also references therein.)

As for *cats*, the experimental realization by the self-modulation hasn't been reported yet. The large interaction times are necessary to produce cats, and the linear absorption makes the observation unlikely [9, 10].

Here, we consider another mechanism which also prohibits the production of cat states by the self-modulation of optical pulses, but has nothing to do with the linear absorption. The cats could be observed in a single mode, if other modes have no need to be taken into account. In this case, this mode can be treated as an Effective Oscillator (EO). We should define correctly this EO and estimate limits of the description of distributed systems in terms of one EO or a few discrete EOs.

A similar analysis could be applied to the case of the detection of the automodulated beam with another automodulated beam. Such a scheme was analyzed [11, 12] in the two-modes approximation. Here we limit the consideration to the simplest scheme shown in Fig. 1.

To simplify, we consider only the single-dimensional case. This case corresponds to a single-mode waveguide (optical fiber). The large-distance propagation of quantum light in a nonlinear dispersive fiber can be described in terms of *quantum solitons* [13]. It was shown [14] that quantum solitons are stable, they have quite definite form [15], and they show the correct classical limit. In the case of high nonlinearity and short-distance propagation, effects of the chromatic dispersion are small in comparison with nonlinear effects. Then, the expansion with the "quantum solitons" should require many terms, and the practical calculations with them are difficult. Note that the same case takes place even in classical nonlinear optics. For example, the book [16] describes the exact solution of

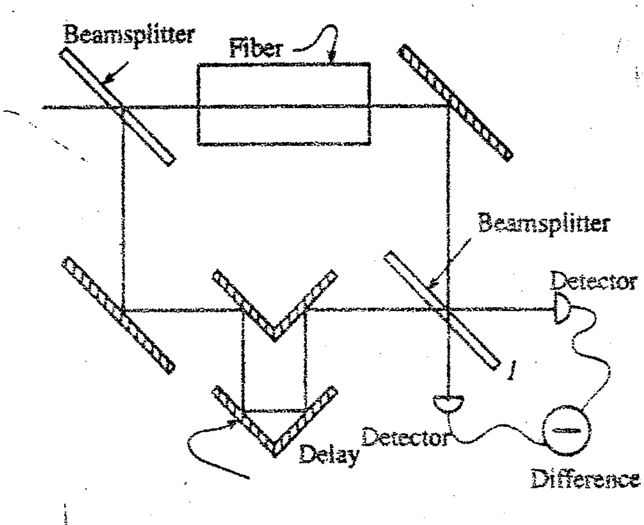

Fig. 1. Scheme of the detection of the dispersion (33) of the operator $\hat{b}$ in the travelling wave.

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the classical nonlinear Schrödinger equation (NSE) in terms of general solitons, but the numerical algorithm supplied there doesn't use this "exact" solution; it is based on the splitting of physical factors.

Here we are interested in the case of high nonlinearity and low chromatic dispersion. We construct the approximate description in terms of EOs. Note that a similar analysis can be applied to the 3-dimensional case, too.

QUANTUM NONLINEAR SCHRÖDINGER EQUATION

The distributed nonlinear interaction of optical pulses in a nonlinear fiber can be described in terms of δ-commuting operators of creation and annihilation of photons in definite point. In quasi-monochromatic approximation, these operators are Fourier-transforms of the operators of creation and annihilation of photons with a definite wavenumber.

To avoid difficulties with pulses of width less than the wavelength, we need to introduce at least a small chromatic dispersion. This leads to the known Hamiltonian [11–15]:

$$\hat{\mathcal{H}} = \frac{\hbar\omega''}{2}\int dz \frac{\partial\hat{a}(z)^{\dagger}}{\partial z}\frac{\partial\hat{a}(z)}{\partial z} + \hbar\chi\int dz\hat{a}^{\dagger}(z)^2\hat{a}(z)^2. \quad (1)$$

The corresponding Heisenberg equation has the form of NSE [10]:

$$i\frac{\partial\hat{a}(z)}{\partial t} = \frac{\omega''}{2}\frac{\partial^2\hat{a}(z)}{\partial z^2} + 2\chi\hat{a}^{\dagger}(z)\hat{a}(z)^2. \quad (2)$$

Values of the constants ω'' and χ can be related with the classical physical parameters. The first one is the classical chromatic dispersion of the fiber, $\omega'' = \partial^2\omega(k)/\partial k^2$, where $\omega(k)$ is the frequency of the normal classical mode with wavenumber k. The nonlinear constant can be expressed as follows:

$$\chi = \frac{24\pi^2}{\hbar}\int\chi^{(3)}L(x_{\perp})^4 dx_{\perp}, \quad (3)$$

where $\chi^{(3)}$ is the classical cubic nonlinearity, and L is the classical normalized transversal mode of the fiber. (We suppose it depends on k slowly and this dependence has no need to be taken into account.)

If we approximate the field operator with a single mode, Hamiltonian (1) becomes

$$\hat{\mathcal{H}}_{\text{local}} = \hbar\kappa(\hat{a}^{\dagger})^2\hat{a}^2. \quad (4)$$

The evolution of the initially coherent state due to Hamiltonian (4) appears as squeezing, as wavepacket diffusion, and as fractional revivals in dependence of the nonlinear constant and the time of interaction [2–5].

It is important to find the relationship between the constants χ which appear in (1), (2) and κ which appears in (4). To do this, let us define the "local" operators of annihilation

$$\hat{a}_m = \int f_m(z)\hat{a}(z)dz, \quad (5)$$

where $\{f\}$ is some complete orthonormal set of functions, i.e.,

$$\int f_m(z)^* f_n(z)dz = \delta_{m,n},$$
$$\sum_n f_n(x)^* f_m(y) = \delta(x-y) \quad (6)$$

These relations lead to

$$\hat{a}(z) = \sum_n f_n(z)\hat{a}_n. \quad (7)$$

Substituting (7) into the Hamiltonian (1), we have:

$$\hat{\mathcal{H}} = \hbar\sum_{m,n}\mathcal{L}_{m,n}\hat{a}_m^{\dagger}\hat{a}_n + \hbar\sum_{j,l,m,n}\mathcal{N}_{jlmn}\hat{a}_j^{\dagger}\hat{a}_l^{\dagger}\hat{a}_m\hat{a}_n, \quad (8)$$

where

$$\mathcal{L}_{m,n} = \frac{\omega''}{2}\int dz\frac{\partial f_m(z)^*}{\partial z}\frac{\partial f_n(z)}{\partial z}, \quad (9)$$

$$\mathcal{N}_{jlmn} = \chi\int dz f_j(z)^* f_l(z)^* f_m(z) f_n(z). \quad (10)$$

If we limit our consideration to a finite number $\mathcal{K}+1$ of modes, each operator $\hat{a}_n$ $(n = 0, \ldots, \mathcal{K})$ can be treated as field operators of an nth EO. In such a way, if we take $\mathcal{K} = 0$, we have the approximation of a single EO. This seems to be a correct way to define EO. Note that the value of the interaction constant $\kappa = \mathcal{N}_{0,0,0,0}$ depends on the form of the principal mode f_0.

This mode may describe the initial form of the pulse; then the initial coherent state of the field can be written as follows:

$$\psi(0) = \exp(\alpha\hat{a}_0^{\dagger} - \alpha^*\hat{a}_0)|0\rangle. \quad (11)$$

We expect that the main effect is the change of the state in the principal mode; the correlation with other EOs should be treated as small, in some sense. To estimate the limits of such an approximation, we should evaluate the number of photons which come into the highest EOs from the principal EO. (Note that the Hamiltonian (8) preserves the total number of photons $\hat{N} = \sum_n \hat{a}_n^{\dagger}a_n$). As such, the most important terms of the Hamiltonian (8) are those that contain a_0 in highest degrees. Thus, we represent the Hamiltonian as $\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_1 + \hat{\mathcal{H}}_2 + \ldots$ where $\hat{\mathcal{H}}_0$ includes all terms which don't exchange the photon number in different EOs, $\hat{\mathcal{H}}_1$ includes terms which allow such an exchange with the principal mode, but are linear concerning highest modes, and $\hat{\mathcal{H}}_2$ includes bilinear and quadratic such terms, etc.:

$$\hat{\mathcal{H}}_0 = \hbar\sum_{n=0}^{\mathcal{K}}(\mathcal{L}_{nn}\hat{a}_n^{\dagger}\hat{a}_n + \mathcal{N}_{nnnn}\hat{a}_n^{\dagger}\hat{a}_n^{\dagger}\hat{a}_n\hat{a}_n) + \hbar\sum_{n<j}^{\mathcal{K}}4\mathcal{N}_{njnj}\hat{a}_n^{\dagger}\hat{a}_j^{\dagger}\hat{a}_n\hat{a}_j, \quad (12)$$

$$\hat{\mathcal{H}}_1 = \hbar\sum_{n=1}^{\mathcal{K}}(\mathcal{L}_{n0}\hat{a}_n^{\dagger}\hat{a}_0 + 2\mathcal{N}_{n000}\hat{a}_n^{\dagger}\hat{a}_0^{\dagger}\hat{a}_0\hat{a}_0 + \text{h.c.}), \quad (13)$$

$$\hat{\mathcal{H}}_2 = \hbar\sum_{n\neq j}^{\mathcal{K}}(\mathcal{L}_{nj}\hat{a}_n^{\dagger}\hat{a}_j + \mathcal{N}_{nj00}\hat{a}_n^{\dagger}\hat{a}_j^{\dagger}\hat{a}_0\hat{a}_0 + \mathcal{N}_{00nj}\hat{a}_0^{\dagger}\hat{a}_0^{\dagger}\hat{a}_n\hat{a}_j + 4\mathcal{N}_{n0j0}\hat{a}_n^{\dagger}\hat{a}_0^{\dagger}\hat{a}_j\hat{a}_0) + \hbar\sum_{n=1}^{\mathcal{K}}(\mathcal{N}_{nn00}\hat{a}_n^{\dagger}\hat{a}_n^{\dagger}\hat{a}_0\hat{a}_0 + \text{h.c.}), \quad (14)$$

and $\hat{\mathcal{H}}_3$, $\hat{\mathcal{H}}_4$ include third and fourth degrees of operators higher modes. (Sometimes, we omit commas as separators of subscripts.)

In the interaction representation, we can represent the wave function as the expansion with the photon states:

$$\Psi(t) = \sum_{\mathbf{l}} C_{\mathbf{l}}(t)\varepsilon_{\mathbf{l}}(t)|\mathbf{l}\rangle, \quad (15)$$

where C are the coefficients, $\mathbf{l} = l_0, l_1, \dots, l_{\mathcal{K}}$ is the vector of the numbers of photons in EOs,

$$\varepsilon_{\mathbf{l}}(t) = \exp(-i\mathcal{E}_{\mathbf{l}}t), \quad (16)$$

and $\mathcal{E}$ are eigenvalues of the Hamiltonian $\hat{\mathcal{H}}_0$:

$$\mathcal{E}_{\mathbf{l}} = \langle\mathbf{l}|\hat{\mathcal{H}}_0|\mathbf{l}\rangle = \sum_{j=0}^{\mathcal{K}}(\mathcal{L}_{jj}l_j + \mathcal{N}_{jjjj}l_j(l_j-1)) + \sum_{j<n}4\mathcal{N}_{jnjn}l_jl_n. \quad (17)$$

Then the Schrödinger equation $i\hbar\frac{\partial\Psi}{\partial t} = \hat{\mathcal{H}}\Psi$ gives the equation for the coefficients:

$$i\hbar\sum_{\mathbf{l}}\frac{\partial C_{\mathbf{l}}(t)}{\partial t}\varepsilon_{\mathbf{l}}(t)|\mathbf{l}\rangle = \sum_{\mathbf{l}}C_{\mathbf{l}}(t)\varepsilon_{\mathbf{l}}(t)(\hat{\mathcal{H}}_1 + \hat{\mathcal{H}}_2 + \dots)|\mathbf{l}\rangle. \quad (18)$$

This equation describes the intermodal photon exchange.

In the following, we neglect terms due to $\mathcal{H}_3$ and $\mathcal{H}_4$: Many of the photons in the highest modes are necessary to make these terms important, while we assume that almost all photons are concentrated in the principal mode.

PERTURBATIONAL THEORY

To investigate the validity of the single-EO approximation, we may use perturbation theory. We should estimate the amount of photons that abandon the principal mode and escape to higher modes. The interaction Hamiltonians $\hat{\mathcal{H}}_1 + \hat{\mathcal{H}}_2$ create photons in the highest modes, annihilating them in the principal one. So, to calculate, for example, the number of "escaped" photons in the second order, we only need the first-order solution of the wave function: The projection of state $a_n^{\dagger}a_n\Psi^{(2)}$ to the $\Psi^{(0)}$ gives zero at $n \neq 0$, where $\Psi^{(0)}$ is the zero-order solution and $\Psi^{(2)}$ is the second-order one.

Expanding the coefficients $C(t) = C^{(0)} + C^{(1)}(t) + \dots$, we get the first-order solution:

$$C^{(1)}_{l,\dots,1_j\dots} = \frac{e^{-it(\mathcal{L}_{00} - \mathcal{L}_{jj} + 2\mathcal{N}_{0000}l - 4\mathcal{N}_{j0j0}l)} - 1}{\mathcal{L}_{00} - \mathcal{L}_{jj} + 2\mathcal{N}_{0000}l - 4\mathcal{N}_{j0j0}l} \times(\mathcal{L}_{j0}\sqrt{l+1} + 2\mathcal{N}_{j000}l\sqrt{l+1})C^{(0)}_{l+1,\mathbf{0}}, \quad (19)$$

$$C^{(1)}_{l,\dots,2_j\dots} = \frac{e^{-it(2\mathcal{L}_{00} - 2\mathcal{L}_{jj} + \mathcal{N}_{0000}(4l+2) - 8\mathcal{N}_{j0j0}l - 2\mathcal{N}_{jjjj})} - 1}{2\mathcal{L}_{00} - 2\mathcal{L}_{jj} + \mathcal{N}_{0000}(4l+2) - 8\mathcal{N}_{j0j0}l - 2\mathcal{N}_{jjjj}} \times\mathcal{N}_{jj00}\sqrt{2(l+1)(l+2)}C^{(0)}_{l+2,\mathbf{0}}, \quad (20)$$

$$C^{(1)}_{l\dots1_j\dots1_n\dots} = \frac{e^{-it(2\mathcal{L}_{00} - \mathcal{L}_{jj} - \mathcal{L}_{nn} + \mathcal{N}_{0000}(4l+2) - 4\mathcal{N}_{0j0j}l - 4\mathcal{N}_{n0n0}l - \mathcal{N}_{njnj})} - 1}{2\mathcal{L}_{00} - \mathcal{L}_{jj} - \mathcal{L}_{nn} + \mathcal{N}_{0000}(4l+2) - 4\mathcal{N}_{0j0j}l - 4\mathcal{N}_{n0n0}l - \mathcal{N}_{njnj}}\mathcal{N}_{nj00}\sqrt{(l+1)(l+2)}C^{(0)}_{l+2,\mathbf{0}}, \quad (21)$$

where $\dots1_j\dots$ means that one photon appears in the jth mode, $\dots2_j\dots$ means two photons in the jth mode, other modes (except the principal one and mentioned ones) contain no photons; $\mathbf{0}$ means the sequence of zeros, repeated $\mathcal{K}$ times. All other coefficients $C^{(1)}$ (which are not mentioned in (19)–(21)) are equal to zero.

With the first-order solution, the number N of photons in the highest modes can be estimated:

$$N = \sum_{l=0}^{\infty}\left(\sum_{j=1}^{\mathcal{K}}\left|C^{(1)}_{l,\dots,1_j\dots}\right|^2 + 2\sum_{j=1}^{\mathcal{K}}\left|C^{(1)}_{l,\dots,2_j\dots}\right|^2 + 2\sum_{j=1}^{\mathcal{K}}\sum_{n=1,n\neq j}^{\mathcal{K}}\left|C^{(1)}_{l,\dots,1_j,\dots,1_n\dots}\right|^2\right). \quad (22)$$

Note that the last double sum is the most dangerous for the single-EO approximation: It contains $\mathcal{K}^2$ terms while the first two are of only $\mathcal{K}$ ones only.

We have no general proof that the amount expressed by (22) is high for the most interesting cases like the Schrödinger cats generation, independently of the choice of the functions f. However, in the next section we make some estimation for the almost-rectangular initial pulse f_0.

NUMERICAL SIMULATIONS

In this section we present an example of the construction of a system of modes for the initial pulse f_0 of the form of trapece.

To simplify the calculus, we consider the incomplete set of modes as an intermediate step between the single-mode approximation and the whole continuum.

Define the principal mode as follows:

$$f_0(x) = \begin{cases} 1/\sqrt{2L+\frac{2}{3}M}, & |x| \le L, \\ (L+M-|x|)/\sqrt{2L+\frac{2}{3}M}, & L \le |x| \le L+M \\ 0, & |x| \ge L+M. \end{cases} \quad (23)$$

At $L \ll M$, such a pulse can be treated as a rectangular one. The advantage of a pulse of this kind is that the change of its form appears mainly at its edges; so, the nonlinear interaction in the Kerr medium causes the rotation in the complex plane rather than the deformation of its profile. But we need $M > 0$ to keep the coefficient $\mathcal{L}_{00}$ finite. Choose some additional modes, let

$$f_k(x) = \begin{cases} \frac{1}{\sqrt{L}}\sin\left(\frac{\pi n}{L}x\right), & |x| \le L \\ 0, & |x| \ge L. \end{cases} \quad (24)$$

Note that this set is not complete, even at the limit $\mathcal{K} \longrightarrow \infty$. At least, the analogous function with cos() would be added; this would double the amount of "lost" photons which abandon the principal mode.

Substitution of (23) and (24) into (9) gives:

$$\mathcal{L}_{00} = \frac{\omega''}{2M(M/3+L)}, \quad (25)$$

$$\mathcal{L}_{nn} = \frac{\omega''}{2}\left(\frac{\pi n}{L}\right)^2. \quad (26)$$

Substituting (23), (24) into (10), we find:

$$\mathcal{N}_{0000} = \chi\frac{2L+2M/5}{(2L+2M/3)^2} \approx \frac{\chi}{2L}, \quad (27)$$

$$\mathcal{N}_{00jj} = \mathcal{N}_{0j0j} = \mathcal{N}_{0jj0} = \mathcal{N}_{j00j} = \mathcal{N}_{j0j0} = \mathcal{N}_{jj00} = \frac{\chi}{2L+\frac{2}{3}M} \approx \frac{\chi}{2L}, \quad (28)$$

$$\mathcal{N}_{jjjj} = \frac{3\chi}{4L}, \quad (29)$$

$$\mathcal{N}_{jnjn} = \frac{\chi}{2L} \quad (j>0,\ n>0,\ j\neq n). \quad (30)$$

One can see that all nonlinear terms are of the same order of magnitude; we have no small parameter in the Hamiltonian.

To illustrate the role of the highest modes, first consider the interaction of only two of them, the principal one and the first one. Taking $\mathcal{K} = 1$, and substituting (27) into the Hamiltonians (12), (14), we get the resulting Hamiltonian in the following form:

$$\mathcal{H} = \frac{\hbar\omega''}{2}\left(\frac{2}{ML}\hat{a}_0^\dagger\hat{a}_0 + \frac{4\pi^2}{L^2}a_1^\dagger a_1\right) + \frac{\hbar\chi}{2L}\Big(\hat{a}_0^\dagger\hat{a}_0^\dagger\hat{a}_0\hat{a}_0 + \frac{3}{2}\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1 + \hat{a}_0^\dagger\hat{a}_0^\dagger\hat{a}_1\hat{a}_1 + \hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_0\hat{a}_0 + 4\hat{a}_0^\dagger\hat{a}_0\hat{a}_1^\dagger\hat{a}_1\Big). \quad (31)$$

This Hamiltonian is very similar to the Hamiltonian of the interaction of two orthogonaly polarized modes described in [17]. Each of these two modes can be treated as an EO. Of course, only two modes cannot adequately represent the continuum of modes which effectively interact in the nonlinear medium; but we can compare the results for the cases of single modes to the case of two modes.

The efficiency of the linear dephasing between modes can be characterized with the parameter

$$r = \left(\frac{4\pi^2}{L^2} - \frac{2}{ML}\right)\frac{\omega''}{2}. \quad (32)$$

First, we are interested with the squeezing. It can be characterized with the minimal dispersion

$$\mathfrak{D}(t) = \min_{\theta=0}^{\pi}(\langle\hat{b}_\theta^2\rangle - \langle\hat{b}_\theta\rangle^2), \quad (33)$$

where $\hat{b}_\theta = \hat{a}_0^\dagger e^{i\theta} + \hat{a}_0 e^{-i\theta}$.

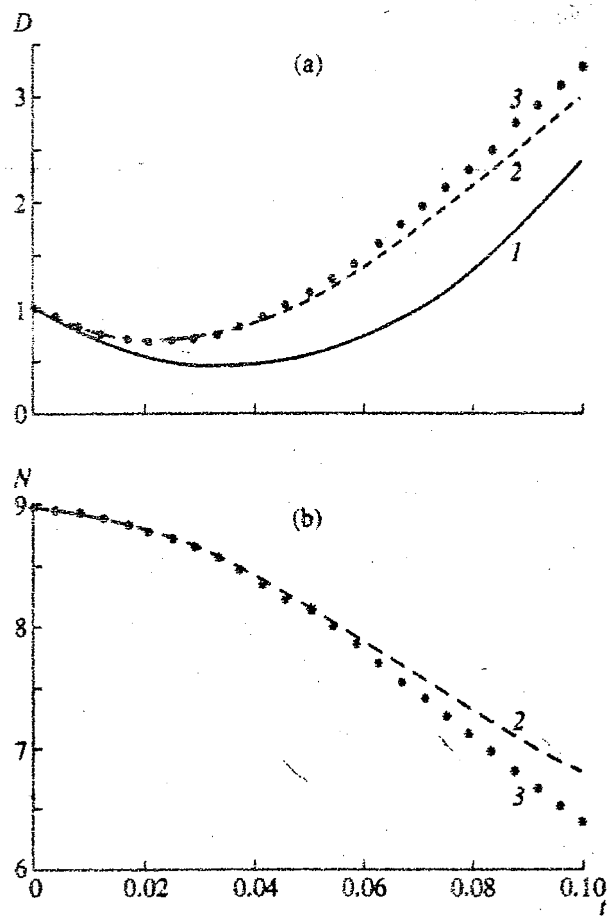

Fig. 2. (a) Evolution of the minimal dispersion (33) in the principal mode for the single-mode approximation (solid curve) and the two-modes approximation with $r = 0$ (dashed curve) and $r = 2$ (asterisks). (b) Evolution of the mean value of the number of photons in the principal mode for the two-modes, cases $r = 0$ (dashed curve) and $r = 2$ (asterisks). Hamiltonian (31), $\chi/2L = 1$. The initial coherent state with $\alpha = 3$.

To simplify formulas in the following, we use the special system of units where $\chi/(2L) = 1$. To compare the case of the single mode to the case of two modes, we made the computer simulation with the initial coherent state $|\rangle = |\alpha,0\rangle$ with $\alpha = 3$. As usual, the Latine index or number at the wave function indicates the photonic state while the Greek letter means the coherent state.

Figure 2a shows the minimal dispersion (33) versus time t of interaction in the single-mode case in comparison with the case of two modes at $r = 0$ and $r = 2$. At these conditions, the minimal dispersion becomes significantly greater due to even one additional mode.

This mode steals photons, and the mean value of the number of photons in the principal mode $\langle \hat{N}_0 \rangle = \langle \hat{a}_0^\dagger \hat{a}_0 \rangle$ becomes lower (Fig. 2b).

At the long-time evolution, we see some kind of revivals, (Fig. 3a), but they are not so perfect as in the single-mode case. As for the mean photon number $\langle \hat{N}_0 \rangle$, we see the fast decay at small values of $t < 0.1$ and then the chaotic oscillations. (As for the slow decay which follows these oscillations, it depends on the step of integration and reduces at its decimation.)

Due to the escape of photons from the principal mode, we see the reduction of the minimal dispersion (33) at the plateaus (Fig. 3a). To characterize the quantum state $|\rangle = e^{-i\hat{H}t}|\alpha, 0\rangle$ better, we plot the quasidistribution function Q defined as

$$Q(\beta) = \sum_m |\langle \beta, m| \rangle|^2. \qquad (34)$$

The evolution of function Q by (34) is shown in Fig. 4 for cases of a single mode (a), two modes at $|r| = 0$ (b) and at $r = 2$ (c); for values $t = 0$ (this distribution is the same for all 3 cases), $t = 0.035$, $t = 0.16$, $t = 0.3$, $t = \pi/2$ and $t = \pi$.

At the small values of time, we see almost no difference in the structure of the uncertainty bodies; in all three cases it looks like a deformed circle. But then, the presence of an additional mode makes the uncertainty body irregular, and deforms the Schrödinger cat states which appeared at $t = \pi/2$. The deformation is more irregular in the last case, ($r = 2$), due to the dephasing of the additional mode.

In contrast with the single-mode case, we see almost no revivals for the two-mode cases. So, we conclude that the aggregation of even few additional modes destroys the "cat" states.

REALISTIC ESTIMATES

Now, consider a more realistic example with many modes, but limit our consideration of the case when the greatest part of the photons is concentrated in the principal mode; then, we may use the perturbation theory considered above.

If we neglect the linear dispersion, as in the simulation above, the amount of photons "escaped" from the principal mode to the highest modes would be large at any finite value of time t of interaction, due to the infinite number of these modes. Practically, the amount of these modes is limited by the chromatic dispersion; different coefficients L cause dephasing of very high modes. Thus, at the finite time of interaction, the number of effectively interacted modes is limited.

The time of interaction cannot be very long because of the spreading of the initial pulse. Such spreading occurs at $\omega''t = L^2$; so, we should satisfy $L^2 \gg \omega''t$. This equation can also be considered the lower limit of the longitude L of the pulse.

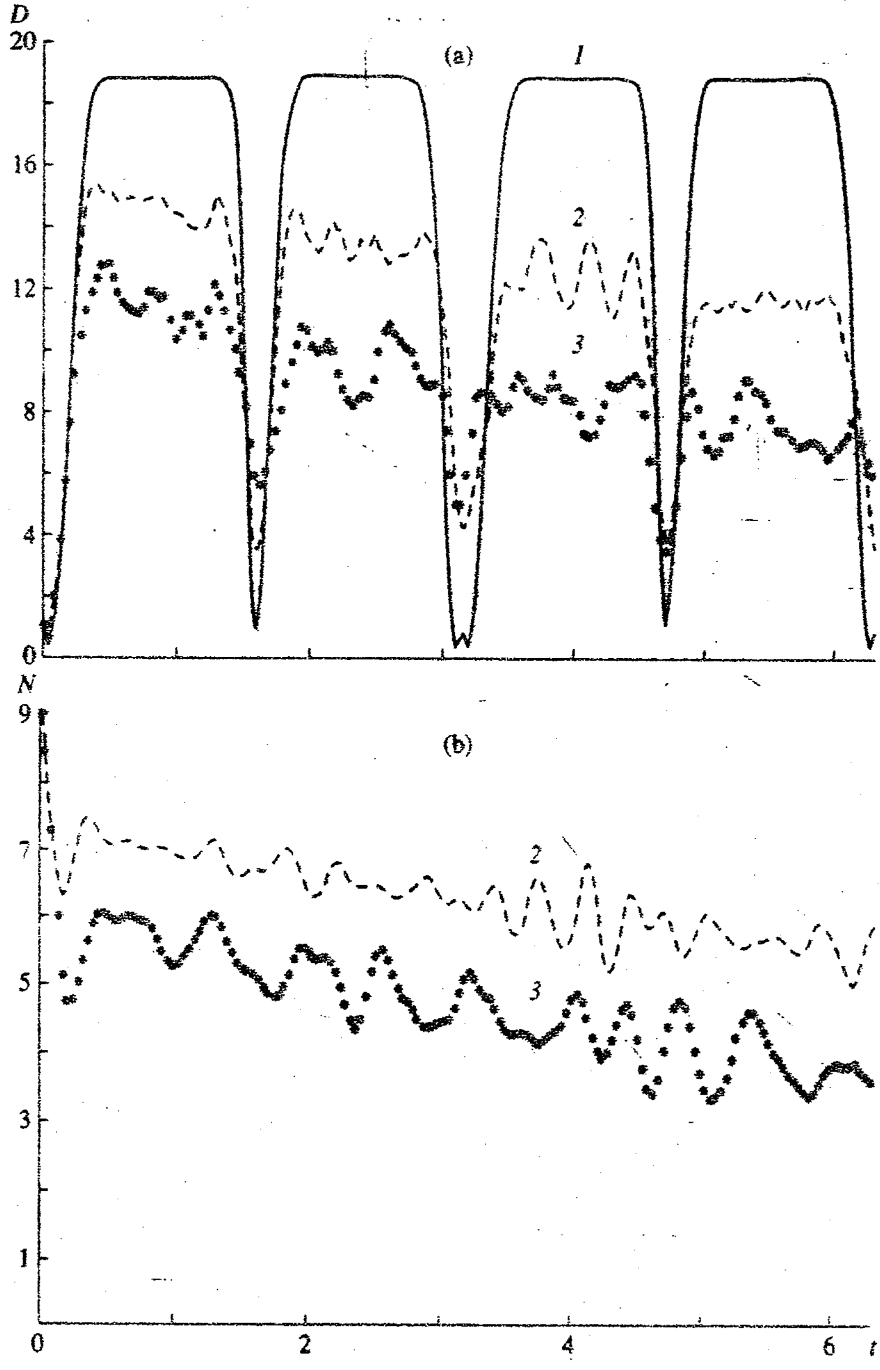

Fig. 3. Lasting evolution of (a) the minimum dispersion (33) and (b) of the mean photon number $\bar{N}_0$ in the fundamental mode for (*1*) the single-mode and two-mode approximation for (*2*) $r = 0$ and (*3*) 2.

The number of modes which are in the resonance during time t is determined by their linear defacing due to exponential factors in (19)–(21). For the rough estimation, let $\mathcal{K}$ be such that

$$\frac{\omega''}{2}\left(\frac{\pi}{L}\mathcal{K}\right)^2 t \approx \pi. \tag{35}$$

This gives

$$\mathcal{K}^2 \approx \frac{2L^2}{\pi\omega'' t}. \tag{36}$$

To estimate the sum in (21), substitute the double sum with $\mathcal{K}^2$. Suppose that the initially coherent state

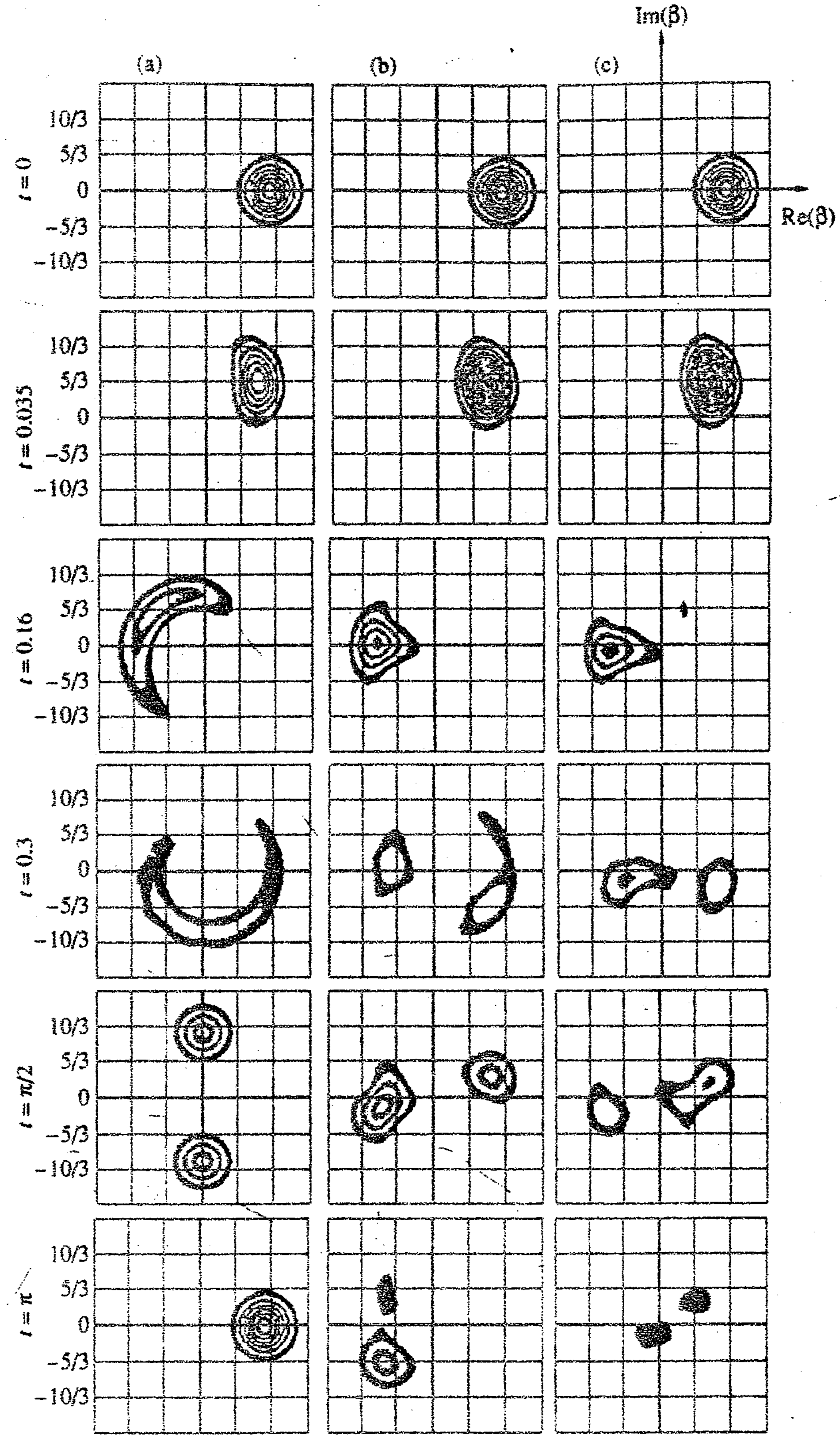

Fig. 4. Quasi-probabilities $Q(\beta)$ for times $t = 0$, $t = 0.035$, $t = 0.16$, $t = 0.3$, $t = \pi/2$, and $t = \pi$: (a) single-mode case, two-mode case for (b) $r = 0$ and (c) $r = 2$.

has the mean photon number l_0, and use this value to estimate the square root in (21). Substituting the first fraction in (22) with it, and taking into account that modes with "cos" discussed above would give the similar amount of "lost" photons, we have the estimation

$$N_{\rm lost} = \frac{32}{\pi}\frac{L^2}{\omega'' t}\mathcal{N}^2 l_0^2 t^2. \qquad (37)$$

Note, that $l\mathcal{N}t = \varphi$ is the classical nonlinear phase caused by the self-interaction. This gives the estimation:

$$N_{\rm lost} = \frac{32}{\pi}\frac{L^2}{\omega'' t}\varphi^2. \qquad (38)$$

If we take $L^2/\omega'' t \approx 10^2$, $l \approx 10^7$ and insist that the relative amount of lost photons is less that 1%, then this gives the estimation of the maximal nonlinear phase shift which can be achieved without strong decoherence of the quantum state:

$$\varphi_{\max} \approx \sqrt{(\pi/32)\times 0.01\times 10^7\times 0.01} \approx 10. \qquad (39)$$

We know that, in the single-mode approximation, the maximal squeezing occurs at $\varphi \approx 1$, while for the cats we need $\varphi = \pi l/j$.

Formula (31) could allow us to achieve high values of the phase φ at high values of l_0. But such values can be realized only at the negative values of chromatic dispersion, $\omega'' < 0$. At positive ω'', the instability of the quasi-monochromatic wave (see, for ex., [18] and references therein) causes the exponential growth of perturbations with the appropriate wavenumber and phase. The increase of the phase for 2π causes the increase of their intensity for $\exp(4\pi)$ times. So, such distortions should cause the destruction of the initial pulse even if these distortions begin with the quantum noise estimated above. The quantitative investigation of the quantum aspect of such instability requires the following investigation.

Note, that parameters of the pulse appear in the final formula (30) only in the dimensionless combinations $L^2/\omega'' t$ (which characterizes the linear spreading of the pulse) and $\varphi = \mathcal{N} l_0 t$ (which characterizes the nonlinear phase shift). This indicates that the formula (30) should have a general sense and be valid for many forms of the initial pulse and any system of basis functions $\{f\}$.

Values in (31) permit one to achieve squeezing, as it was indicated in [18, 19] and observed in [8], but no Schrödinger cats can be realized in the travelling wave by self-modulation, even without absorption, independently on the signs of χ and ω''.

Such a limit is more fundamental than the disaggregation of "cats" due to the absorption (or amplification) discussed in [10] and considered in more detail in [20].

CONCLUSIONS

We formulated the correct definition of the EO. Such a definition allows one to consider nonlinear interactions of travelling waves out of the single-EO approximation and estimate the region of validity of the single-EO approximation in each special case.

Such formalism is applied to the problem of the quantum state of the self-modulated optical pulse. It is shown that the single-EO approximation becomes invalid at large values of the nonlinear phase due to the escape of photons from the principal mode to highest modes. Maximal values of the nonlinear phase are given by formula (30). For example, at 10^7 photons/pulse a nonlinear phase about 10 can be achieved at a relatively small amount of escaped photons. (At $\omega'' > 0$, the maximal phase is also limited with the instability of monochromatic wave.) At high (greater than unity) values of the nonlinear phase, the exponential growth of the number of escaped photons can make this limitation even stronger.

Such an estimation limits the maximal value of squeezing which can be obtained by the self-modulation of optical pulses of a given frequency and energy. Also, this estimation prohibits the creation of the Schrödinger cat states from the coherent states in such a way.

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