Slab Delivery of Incoherent Pump Light to Double-Clad Fiber Amplifiers: Numerical Simulations

Patrick Kano, Dmitrii Kouznetsov, Jerome V. Moloney, and Moysey Brio

***Abstract*—Compact fiber amplifiers with tapered slab delivery of the pump are analyzed numerically. A procedure for the selection of the structural parameters of this device is presented together with a simple asymptotic estimate for the efficiency. Simulations based on the scalar paraxial beam propagation equation demonstrate the validity of this estimate.**

***Index Terms*—Double-clad fiber, optical fiber amplifiers, scalar paraxial beam propagation equation.**

I. Introduction

Efficiently pumping a fiber over a distance of a few centimeters is a technical problem of current interest. A tapered slab geometry, Fig. 1, has recently been proposed as a compact double-clad fiber amplifier design [1]–[5]. Asymptotic estimates for the absorption efficiency and the minimum slab length for reasonable values of the structural parameters have also been suggested [6]. These estimates, however, are based on strong assumptions and require numerical consideration. In this paper, we choose the structural parameters based on the criteria in [6], numerically simulate the device using the scalar beam propagation equation, investigate the accuracy of the estimate for the efficiency, and suggest a domain of validity.

II. Structure Parameters and Asymptotic Estimates

The selection of parameters which are both physically realistic and provide efficient pumping is clearly a crucial step in the device's construction. The following presents the choice of parameters for the structure numerically analyzed in this paper.

First to consider is the choice of fiber. The core and fiber radii and the indices of refraction of the core, fiber cladding, and surrounding coolant should allow for single mode propagation of the signal in the core. This requires that the normalized frequency of the fiber be less than 2.4 [7]. We choose the values summarized in Table I.

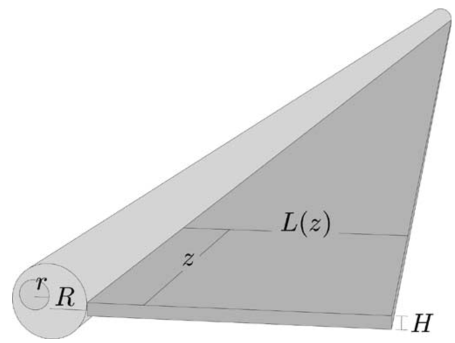

Fig. 1. Tapered slab pumped double-clad fiber.

TABLE I
FIBER PARAMETERS

r_{core}	R_{fiber}	n_{coolant}	n_{cladding}	n_{core}
10 μm	30 μm	1.330	1.560	1.561

The index of refraction of the coolant corresponds to a value near that for the organic liquid coolants often used for double clad devices. The core and cladding have indices typical of silica glasses. The imaginary part of the core index of refraction, which is related to the absorption, remains a parameter. To select a wavelength for the signal, we consider an Er/Yb core [8]. This material emits predominantly at 1.535 μm. The center of the core is offset from the center of the fiber, 9 μm up and 1 μm left. For these parameters, $V \approx 2.287$.

Next, one must choose a source and slab. Since erbium/ytterbium codopants in a phosphate glass have a maximum absorption $\alpha \approx 10$ cm^{-1} near $\lambda = 0.98$ μm, we select a source with this wavelength. If one assumes to have a perfectly matched single mode slab pump in the y-direction, then the number of modes generated by the pump is approximately given by the formula

$$N_{\text{Pump}} = 2\frac{(\text{NA})L_0}{\lambda} \qquad (1)$$

where (NA) is the numerical aperture of the pump in the x-direction [6]. For a slab of initial width $L_0 = 100$ μm, thickness $H = 6$ μm, and an $\text{NA} = 0.15$, the number of modes created by this pump is $N_{\text{Pump}} \approx 30$.

One must also compute the slab index of refraction required so that the lowest order modes of the fiber have strong overlap

Manuscript received February 19, 2004; revised May 6, 2004. This work was supported by the Air Force Office of Scientific Research under Grant F49620-02-1-0380.

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Digital Object Identifier 10.1109/JQE.2004.833232

TABLE II
SLAB AND PUMP PARAMETERS

L_0	H	n_{slab}	N_{Pump}	λ
100 μm	6 μm	1.567	30	0.98 μm

TABLE III
SLAB LENGTHS AND Λ FOR VARIOUS ABSORPTION COEFFICIENTS

α (cm^{-1})	T_0 (cm)	$\Lambda_{\text{clad core}}$ (cm)	$\Lambda_{\text{unclad core}}$ (cm)
1	$6/\pi \approx 1.9099$	10.9099	2.9099
2	$3/\pi \approx 0.9549$	5.4549	1.4549
4	$3/(2\pi) \approx 0.4775$	2.7275	0.7275
8	$3/(4\pi) \approx 0.2387$	1.3637	0.3637
16	$3/(8\pi) \approx 0.1194$	0.6819	0.1819

with the highest order modes of the slab at $z = 0$. In [6], this condition has been shown to reduce to the statement

$$n_{\text{slab}} = \frac{n_{\text{cladding}}}{\left(\frac{\cos \text{NA}}{n\text{cladding}}\right)}. \quad (2)$$

For the present case, $n_{\text{slab}} \approx 1.567$. Table II provides a summary of the slab and pump parameters.

The final step is to determine the tapered slab length T in the estimate of the absorption efficiency for $z < T$ [6]

$$\eta = 1 - \frac{\|E(z)\|^2}{\|E(0)\|^2} = 1 - \left(1 - \frac{z}{\Lambda}\right)^{\frac{\alpha\pi r^2}{HL_0}T} \quad (3)$$

where

$$\Lambda = T\frac{\pi R^2 + HL_0}{HL_0}. \quad (4)$$

The absorption rate of the core α is given by the equation $(d/dz)\|E\|^2 = -\alpha\|E\|^2$, where $\|E\|^2 = \int_\Omega |E|^2 dxdy$. The parameter Λ can be interpreted as an effective absorption length.

In order to allow for a uniform intensity of the pump in the fiber, the length of the slab is chosen so that the exponent in the efficiency is one [6]

$$T_0 = \frac{HL_0}{\alpha\pi r^2}. \quad (5)$$

Finally, if one defines

$$\alpha_{\text{effective}} \equiv \alpha\frac{\pi r^2}{\pi R^2 + HL_0} \quad (6)$$

the the expressions for Λ and η can be written concisely

$$\Lambda(T_0) = \frac{1}{\alpha_{\text{effective}}} \quad (7)$$

$$\eta(z) = z\alpha_{\text{effective}} \quad (8)$$

The efficiency is thus predicted to be directly proportional to the propagation distance. Obviously, the fiber can be extended longer than T_0 in order to use the part of the pump which is not absorbed but still guided. In Table III are the tapering length and approximate $\Lambda(T_0)$ for physically relevant values of α and the parameters given above.

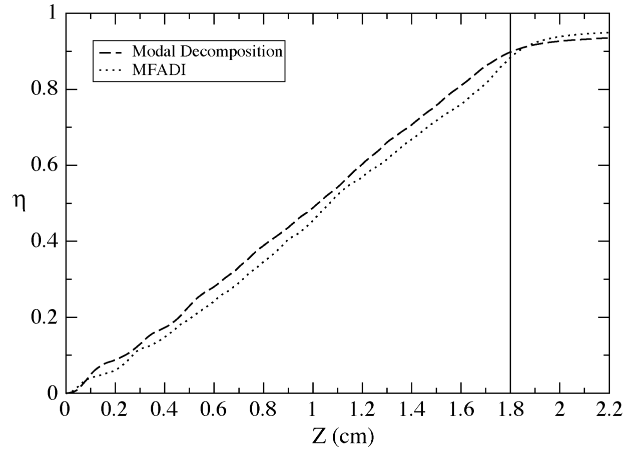

Fig. 2. Efficiency η versus propagation distance measured from simulations using the MFADI (dotted line) and modal decomposition (dashed line) codes.

III. NUMERICAL COMPUTATIONS AND DISCUSSION

A. Scalar Paraxial Beam Propagation Method

The scalar paraxial beam propagation equation

$$\frac{\partial E}{\partial z} = \frac{1}{2i\beta}\left[\nabla_\perp^2 - \left(\beta^2 - k_0^2 n^2\right)\right]E \quad (9)$$

where β is the propagation constant, k_0 is the free space wave number, $n = n(x, y, z)$ is a complex index of refraction, and E is the envelope to one transverse component of the electric field, and is a standard tool for the numerical simulation of photonic structures [9]–[11]. Here, this equation is solved by the split-step Mitchell–Fairweather alternating direction implicit (MFADI) method [12], [13]. Perfectly reflecting boundary conditions are implemented to conserve the total energy in the computational domain. The propagation constant β is adaptively determined by [14]

$$\beta^2 = \frac{\int_\Omega |k_0 nE|^2 - |\nabla_\perp E|^2 dxdy}{\int_\Omega |E|^2 dxdy}. \quad (10)$$

One test of the code has been to simulate the tapered slab coupled to an unclad core studied in [6]. The indices of refraction are $n_{\text{slab}} = 1.58$, $n_{\text{core}} = 1.56$, $n_{\text{coolant}} = 1.54$. The core has a 4.3-$\mu$m radius and a local absorption coefficient of 10 cm^{-1}. The slab has a 6-μm thickness, 1-cm initial width, and a length of 1.8 cm.

The initial condition utilized to model the diode pump consists of an incoherent combination of slab modes in the x-direction with a single mode in the y-direction [5]. In particular, $E(x, y) = \mathcal{X}(x)\mathcal{Y}(y)$, where

$$\mathcal{X}(x) = \begin{cases} \sum_m \sin\left(\frac{\pi m}{L_0}\left(x + \frac{L_0}{2}\right)\right)e^{(i\phi_m)}, & -\frac{L_0}{2} \le x \le \frac{L_0}{2} \\ 0, & \text{else} \end{cases} \quad (11)$$

$$\mathcal{Y}(y) = \begin{cases} \cos\left(\kappa\frac{H}{2}\right)e^{\sigma\left(\frac{H}{2}+y\right)}, & y < -\frac{H}{2} \\ \cos(\kappa y), & -\frac{H}{2} \le y \le \frac{H}{2} \\ \cos\left(\kappa\frac{H}{2}\right)e^{\sigma\left(\frac{H}{2}-y\right)}, & y > \frac{H}{2} \end{cases} \quad (12)$$

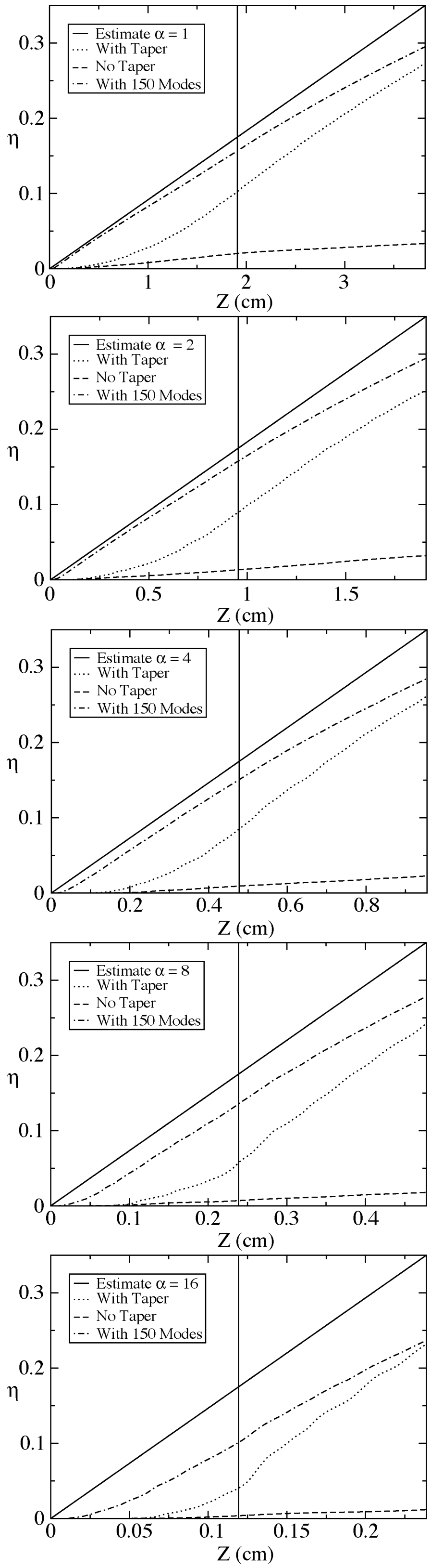

Fig. 3. Absorption efficiency η for $\alpha = 1, 2, 4, 8, 16$ cm^{-1}. The estimated efficiency using (8) is indicated by a solid curve.

where x, y refer to the coordinate system with its origin at the center of the lateral cross section of the slab. The phase $\phi_m \in [-\pi, \pi]$ for each mode ensures that the initial condition

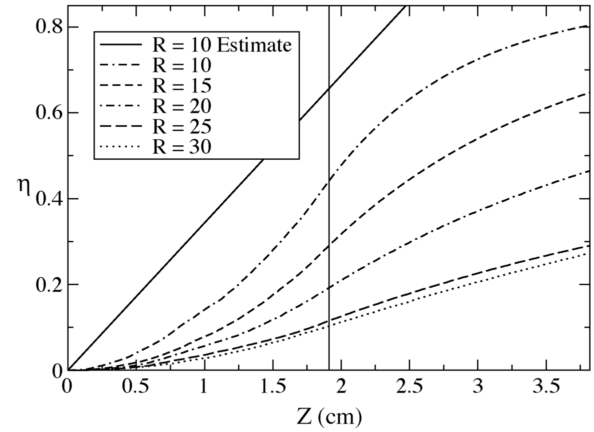

Fig. 4. Absorption efficiency η versus the fiber radius R for $\alpha = 1$.

is random as it would be from a diode pump. The coefficients κ and σ solve the system of equations

$$\kappa^2 + \sigma^2 = k^2 \left(n_{\text{slab}}^2 - n_{\text{coolant}}^2\right) \tag{13}$$

$$\tan\left(\kappa \frac{H}{2}\right) = \frac{\sigma}{\kappa}. \tag{14}$$

Finally, to emulate the effect of tilting the light toward the fiber, the initial field is multiplied by the phase factor $e^{ip_0(x+(L_0/2))} = e^{i(0.2(\mu\text{m})^{-1}(x+(L_0/2)))}$. This corresponds to a horizontal velocity $v_x = (-p_0/\beta)$ and an inclination of the pump with respect to the core of approximately 1.3°. Fig. 2 shows the evolution of the efficiency averaged over many realizations of the initial conditions. The vertical line indicates the length of the tapered slab. The agreement between the previous computations [6] based on a modal decomposition and those from the MFADI method is clear.

B. Comparison and Results

In Fig. 3 are results from simulating the structure with the parameters in Section II and the initial conditions in III-A. The structure with a tapered slab is clearly more efficient than the one with a slab which is not tapered. As anticipated, the rate of absorption is approximately linear and has a slope near that of $\alpha_{\text{effective}}$. The estimate is, however, considerably more than that measured, especially for larger values of α. The discrepancy in the absorption can be attributed to two sources. One is the thickness of the cladding. With cladding, the pump light requires additional distance to diffuse through the fiber before it can be absorbed in the core. As seen in Fig. 4, reducing the thickness of the cladding improves the absorption efficiency. An additional cause for the delay is the lack of sufficient high order modes in the initial condition. The estimate proposed in [6] is predicated on the assumption that the slab and fiber modes have strong overlap and that the highest order modes of the slab are quickly expelled as the width of the slab is decreased. If the initial condition contains an insufficient number of higher order modes then there occurs a delay in the transfer of light from the tapered slab to the fiber until the resonance condition is fulfilled. Simulating an initial condition where the number of modes is increased beyond the suggested number of $N_{\text{Pump}} = ((\text{NA})L_0k_0/\pi) \approx 30$ modes delivers the improvement seen in Fig. 5. From the simulations in (3) it

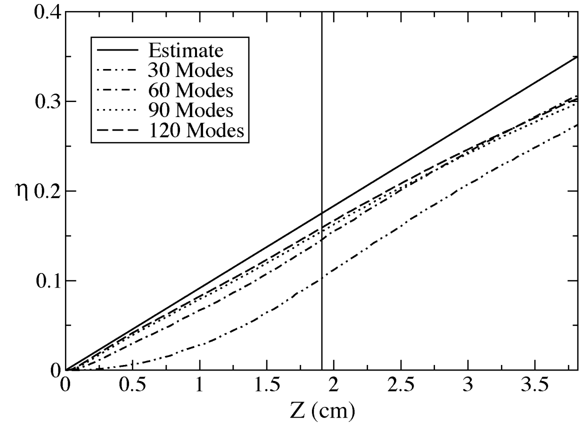

Fig. 5. Absorption efficiency η versus initial condition modes for $\alpha = 1$.

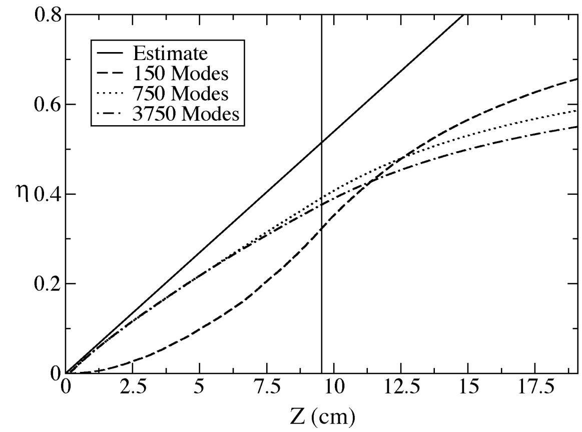

Fig. 6. Absorption efficiency η for slab width 500 μm and $\alpha = 1$.

is also clear that increasing the number of modes to 150 has the best effect for small values of α. Finally, in Fig. 6 we see that the behavior is reproduced in a wider slab with a 500-μm width. For this case, $T_0 \approx 9.5493$ cm, $\Lambda_0 \approx 18.5493$ cm and all other parameters are identical to the 100-μm width slab. Again, the efficiency is approximately linearly increasing and shows better agreement with the estimate for a more random initial condition.

IV. Conclusions

The tapered slab geometry is promising as a compact and efficient pump design. The simulations show that the linear efficiency estimate suggested in [6] is accurate over a few centimeters for small values of $\alpha < 5$ cm^{-1} and highly random initial conditions. The delay in the absorption observed for initial conditions with few modes can be overcome by adding higher order modes. Greater efficiency can also be achieved by reducing the thickness of the fiber cladding. The simplicity of the expression for the efficiency makes it an attractive tool for designing tapered slab pumps. An experimental investigation of the tapered slab geometry is suggested as a source for future work.

Acknowledgment

The authors would like to thank M. Kolesik, A. Mafi, G. Katona, M. Mansuripur, V. Temyanko, and Nasser Peyghambarian at the Arizona Center for Mathematical Sciences and the Optical Science Center, University of Arizona, Tuscon, for stimulating discussions.

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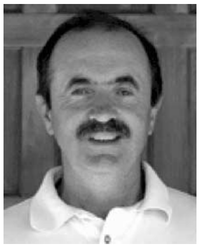

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