A PHASE RETRIEVAL ALGORITHM BASED ON ANALOGY FOR OPTICAL IMAGE PROCESSING

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An optical scheme is proposed for the reconstruction of the phase distribution of the light field from two amplitude distributions. A mathematical algorithm is developed, which models the operation of the scheme. Numerical calculations revealed the rapid convergence of the algorithm.

A number of mathematical algorithms is known which allow the complex light to be reconstructed using two spatial distributions of the field modulus ("two-moduli problem") [1–3]. Here we propose a field reconstruction method which could be realised with the use of an optical arrangement.

Let us assume that the initial monochromatic field (carrying information on a given object) was recorded in two planes: image plane and Fourier-plane. We express the field in the image plane by $E(x)=a(x)\exp(\mathrm{i}\varphi(x))$ and in the Fourier-plane by $F(x)=A(x)\exp(\mathrm{i}\Phi(x))$,

$$F(x)=(1/\sqrt{2\pi})\int E(x')\exp(-\mathrm{i}xx')\,\mathrm{d}x'\equiv LE.$$

Function $E(x)$ is supposed to have a finite support. Suppose two transparencies are produced, with their transmittance corresponding to the functions $a(x)$ and $A(x)$. Fig. 1 shows a one-directional ring laser in which lens 4 provides the Fourier-transformation from plane 1 to 1′ and lenses 5–7 provide the inverse Fourier-transformatioin from plane 1′ to 1. All the lenses have focal length f, the optical path length in the cavity round-trip is $8f$. The above mentioned transparencies must be placed in planes 1 and 1′. To compensate for the losses of the transparencies, two amplifying elements 3 and 3′ (both having the gain coefficient β) are used. These amplifiers must be homogeneous enough and they should have sufficient angular apertures, i.e. they must meet the requirements for the brightness amplifiers in optical systems [4]. Among the existing active media, the copper-vapor laser medium is the most suitable. The low density and the homogeneity of metal vapor ensures the high optical quality of this medium. Besides, the copper vapor has a high gain coefficient (about 1 cm^{-1}). Therefore in a reasonable sized laser tube a high total gain and a wide angular aperture could be achieved simultaneously. These properties allowed the employment of the copper-vapor laser in a laser projection microscope, and the laser introduced no noticeable distortion in the image (see, in particular, ref. [5]). In front of each transparency a nonlinear absorber must be placed. The absorbers are intended to smooth the intensity of the wave incident on the transparency, leaving its phase distribution unchanged. I.e. they should behave like darkening filters. If there is a two-photon absorber

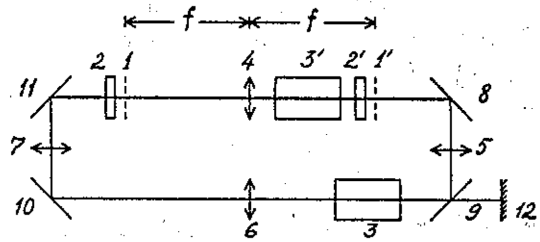

Fig. 1. Optical arrangement for reconstruction of the light field phase function. 1,1′ – transparencies; 2,2′ – nonlinear absorbers; 3,3′ – amplifying elements; 4–7 – lenses; 8–11 – mirrors; 12 – nonreciprocal element.

with a thickness l and if the diffraction effects are negligibly small within l, then we can use the equation

$$dy/dz = -\tfrac{1}{2}(\alpha/l)|y|^2 y,$$

and obtain that the incident field pattern y is transformed at the absorber output into $N(y)$, where

$$N(y) = y(1+\alpha y y^*)^{-1/2}. \qquad (1)$$

Now, the transformation of the field y_n into y_{n+1} in the cavity after a single round-trip can be described by the equation

$$N_{n+1} = \beta L^{-1} A N(\beta L a N(y_n)). \qquad (2)$$

It should be noted that in eq. (1) we can always assume $\alpha = 1$, because the substitution of α is equivalent to the renormalization of the amplitude y, and in eq. (2) all operators except N are linear.

One could take into consideration another mechanism of nonlinear losses, e.g. losses due to the second harmonic generation or to the stimulated Raman scattering. In practice it would be more convenient to make use of the gain saturation of an amplifying element. To this end the length of the amplifier must be less than the diffraction length. In that connection we remind that the copper-vapor lasers having an active gain medium as small as 20 cm are described elsewhere [6].

For the sake of a simpler description we shall suppose in this text that the amplification and the nonlinearity act separately. So we shall use below eq. (2).

Numerical calculations were made using eq. (2). The object field was produced using a random number generator and a procedure of smoothening along the x-axis. The complex function $E(x)$ was assumed to have a two-dimensional gaussian probability distribution in each sampling point. The Fourier pattern $F(x)$ was constructed from the field $E(x)$. We remind that only the moduli $a = |E|$ and $A = |F|$ were used in the calculations. Another random field was used as a trial function $y_0(x)$. We choose the initial intensity to be small, $|y_0|^2 \ll 1$. Therefore at the first stage the transformation of the field was linear, i.e. $N(y) \approx y$. We have no proof of the algorithm convergence. Here the situation is just the same as in the case of the Gerchberg-Saxton algorithm. However, our experience suggests that the algorithm (2) always converges: for every object function we tested and every trial function, a rapid convergence of the algorithm was obtained.

Two regimes could be found during the iteration. If the gain coefficients did not compensate for losses, the field intensity decreased from step to step and the transformation remained linear throughout the whole iteration process. If, on the contrary, the gain was greater than the losses, the "generation" regime was achieved. Here, the transformation of the field was linear only at the very beginning of the iterations and later the intensity increased so much that the nonlinear absorption switched on. Then the intensity growth retarded, the mean value of intensity approximated to the steady-state level, and the final spatial field distribution was achieved. The linear ("damping") regime was described in ref. [7]. In this regime the convergence of the iterative procedure was observed for all object functions that were tested, and it was independent of the initial trial function. The resulting function $y(x) = \lim_{n\to\infty} y_n(x)$ demon-

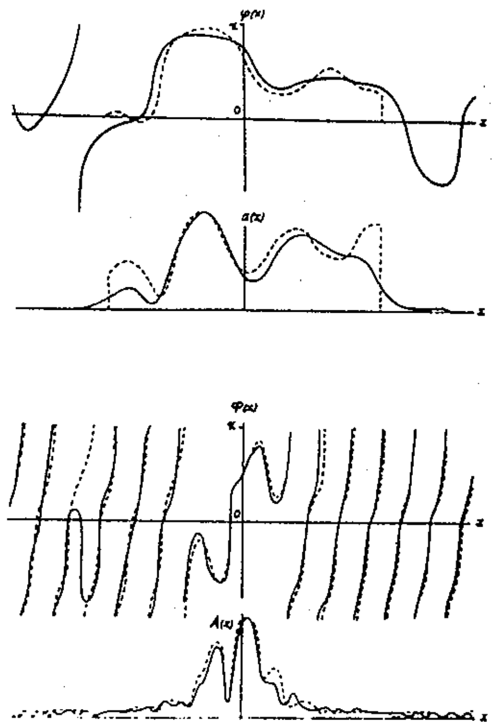

Fig. 2. The dependence of the field phase and amplitude on the transversal coordinate in the image plane (a, b), and in the Fourier-plane (c, d). Dashed lines refer to the object field, and solid lines to the reconstructed field.

strated a resemblance of the object function $a(x)$ $\exp(i\varphi(x))$. The quality of the reconstruction was characterized by the overlap integral

$$\eta = \langle ya \exp(-i\varphi) \rangle \left(\langle yy^* \rangle \langle a^2 \rangle \right)^{-1/2} \quad (3)$$

(reconstruction coefficient). Angular brackets denote the averaging with the spatial coordinate x.

The range of the values η was 0.75–0.90. The rapidity of the convergence was practically independent of the trial function $y_0(x)$.

In the "generation" regime the quality of the reconstruction was much better. During the linear stage, as a rule, a good approximation to the object field was obtained, which was further improved during the nonlinear stage. The procedure took 10–15 steps (including both linear and nonlinear stages). Fig. 2 illustrates the results of the calculations ("generation" regime). It presents the field produced after 10 iterations. This field and the field produced after 9 iterations coincide with a relative accuracy of 10^{-3}. The mean intensity value yy^* in this case was ~ 4, the reconstruction coefficient $\eta \sim 0.96$. In the "damping" regime we obtained $\eta = 0.78$ for the same object field.

It should be noted that in the case $\beta \gg 1$, algorithm (2) turns into the Gerchberg-Saxton algorithm [1]. We also tried the unperturbed Gerchberg-Saxton algorithm, but its convergence was, as a rule, less rapid than that of algorithm (2).

The efficiency of the iterative algorithms should be dependent on the characteristics of the object field. As the object field becomes more complicated (with higher Shannon number) the convergence of the algorithm (2) is retarded and the quality of the reconstruction decreases. The efficiency of the algorithm (2) can be optimized by suitable selection of the parameters β and $\langle y_0 y_0^* \rangle$.

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