Quantum fluctuations do not destroy an optical soliton

D. Yu. Kuznetsov
P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, 117924, Moscow

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The behavior of an optical soliton in a dispersive nonlinear fiber is discussed. The evolution of the fourth-order correlation function is analyzed. This function is independent of the time. The result is formulated as a theorem regarding the correlation function, which also applies to other quantum systems. The fact that this correlation function does not change proves that a soliton is stable. Physical consequences of this stability are discussed.

The quantum-mechanical problem of the propagation of quasimonochromatic light pulses through dispersive nonlinear fibers can be reformulated in terms of a one-dimensional gas with a local interaction of particles (see, for example, Refs. 1 and 2 and the bibliography there). In a sense, this problem can be solved exactly. Belinskiĭ[3] has constructed profiles of the expectation value of the photon density along the coordinate at various times. It was concluded from the spreading of this function that "quantum fluctuations" annihilate an optical soliton. In the present letter we analyze that assertion and study the stability of a quantum soliton.

We must first eliminate some confusion in terminology. If the state of a free particle is described by a wave packet, the width of this packet will sooner or later begin to increase. In this case should we say that "the particle is annihilated?" Common sense suggests that we should not: Otherwise, no quantum-mechanical particle could be regarded as stable. The "annihilation" of a system means that the parts of the system move away from each other by distances which increase without bound as time elapses. For example, a molecule is annihilated if dissociation occurs. To speak of the annihilation of a soliton in exactly the same way would be legitimate only if the photons of which the soliton consists could be detected far apart from each other. Accordingly, in analyzing the question of whether a quantum soliton is destroyed we need to examine the correlation functions which characterize the relative positions of the photons in the soliton.

The light in a nonlinear fiber with a dispersion is described by the quantum Hamiltonian

$$H = \int \hat{\phi}_x^+(x)\hat{\phi}_x(x)\,dx + 2\pi c \int \hat{\phi}^+(x)\hat{\phi}^+(x)\hat{\phi}(x)\hat{\phi}(x)\,dx, \tag{1}$$

with the δ-function-commuting field operators $\hat{\phi}$; here $c = \text{const}$ (Refs. 1 and 3) and $\hbar = 1$. Exact solutions of the Schrödinger equation in the form of eigenfunctions of the Hamiltonian and the momentum are described in Ref. 2:

$$|n,p,t>=\exp(-iE(n,p)t)|n,p>; \quad E(n,p)=np^2-\frac{c^2}{12}n(n^2-1);$$

$$|n,p>=\frac{1}{\sqrt{n!}}\int f_{n,p}(x_1...x_n)\hat{\phi}^+(x_1)...\hat{\phi}^+(x_n)|0>d^nx; \tag{2}$$

$$f_{n,p}(x_1...x_n)=N_n\exp(ip\sum_i x_i+\frac{c}{2}\sum_{i<j}|x_i-x_j|); \quad N_n=\sqrt{\frac{(n-1)!}{2\pi}}|c|^{(n-1)/2}.$$

A soliton solution with a definite phase and a definite amplitude is constructed in the case $c<0$ as a linear combination of solutions of this type:[2,3]

$$|\psi_s>=\sum_{n=1}^{\infty}a_n\int g_n(p)|n,p,t>dp. \tag{3}$$

The particular form of the coefficients a and g is unimportant. The only point utilized below is the circumstance that quantum soliton (3) is constructed from bound states $|n, p, t\rangle$.

The behavior of the matrix element

$$<\psi_s|\hat{\phi}^+(x)\hat{\phi}(x)|\psi_s> \tag{4}$$

was discussed in Ref. 3. A "spreading" of this matrix element means that the spatial localization of the soliton becomes progressively worse. However, this matrix element does not characterize the relative positions of the photons. To see whether a soliton is stable, we need to examine the correlation function

$$K(z)=\int<\psi_s|:\hat{I}(x)\hat{I}(x+z):|\psi_s>dx. \tag{5}$$

Here $\hat{I}(x)=\hat{\phi}^+(x)\hat{\phi}(x)$. It can be shown that $K(0)$ and $\int K(x)dx$ do not depend on the time.[4] This circumstance does not, however, mean that the other moments,

$$x^m(t)=\int K(x)x^m dx \tag{6}$$

cannot vary. In particular, it does not mean that these other moments cannot increase with the time. We will now show that the moments in (6) are integrals of motion. To do this, we substitute (5) into (6) and introduce a coordinate system moving with the soliton. We set $z=y-x$. From (2) and (3) we then find

$$x^m(t)=\sum_n|a_n|^2\int e^{in(p^2-q^2)t}g_n^*(p)g_n(q)M_n^2(p,q)\,dp\,dq, \tag{7}$$

where

$$M_n^m(p,q)=\frac{|c|^{n-1}}{2\pi n}\int(x-y)^m\,dx\,dy\,dx_1...dx_n\,dy_1...dy_n$$

$$\times \exp(-ip\sum_j x_j + iq\sum_j y_j + \frac{c}{2}\sum_{j<k}|x_j - x_k| + \frac{c}{2}\sum_{j<k}|y_j - y_k|)$$
$$\times < 0|\hat{\phi}(x_1)...\hat{\phi}(x_n)\hat{\phi}^+(x)\hat{\phi}^+(y)\hat{\phi}(x)\hat{\phi}(y)\hat{\phi}^+(y_1)...\hat{\phi}^+(y_n)|0> . \quad (8)$$

The commutation relations make it possible to simplify (8):

$$M_n^m(p,q) = \frac{n!}{2\pi}|c|^{n-1}\int(x_1 - x_2)^m\, dx_1...dx_n \exp(-i(p-q)\sum_j x_j + c\sum_{j<k}|x_j - x_k|). \quad (9)$$

The introduction of new integration variables, including $X_0 = \Sigma_j x_j$, shows that (9) is proportional to $\delta(p-q)$. This result rules out a t dependence of (7). All the moments in (6) are therefore constants. We draw the further conclusion that correlation function (5) is itself independent of the time.

The proof does not depend on the particular functions $E(n,p)$ and $g_n(p)$. We thus have a theorem regarding the correlation function: An intensity correlation function does not change in the course of the evolution of a superposition of eigenfunctions of the number of particles, of the Hamiltonian, and of the momentum if the eigenvalues of the Hamiltonian are determined unambiguously by the eigenvalues of the number of particles and of the momentum.

The quantum stability of an optical soliton indicates that the soliton is not annihilated; all that happens is that the quantum uncertainty regarding the coordinate of the soliton increases. The only thing that "spreads out" is the distribution along this coordinate. It is thus clear what we could expect from measurements of the phase and amplitude of a soliton which has "spread out" in accordance with Ref. 3. A detection of some of the photons making up the soliton would result in a reduction of the wave packet. The remaining photons would form a light pulse or packet with a well-defined field amplitude. The theorem regarding the correlation function thus forbids the detection of groups of photons which are far apart from each other and which could be interpreted as the products of an annihilation of the soliton.

Conclusion. The intensity correlation function in (5) does not depend on the time. Consequently, the distribution of distances between photons in a soliton, (3), is conserved. A soliton is stable; "quantum fluctuations"[3] do not annihilate it. The spreading of the matrix element in (4) means no more than an increase in the quantum uncertainty regarding the position of the center of mass of the soliton. The theorem regarding the correlation function may be of independent importance.

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